Fractional Order Discrete States-Space System
Simulink Toolkit User Guide.

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Abstract
This paper presents User Guide for the Fractional Order Discrete
States-Space System Toolkit. The installation procedure, blocks descrip-
tion and examples is presented.

Keywords: discrete fractional systems, fractional Kalman filter, state feedback
control

1 Installation

To install the toolkit unpack the file fastX_Y.zip, where X_Y is a version of
the toolkit, and add the directory to the Matlab path. The zipped file contains
following files:

  * fsim.c - C-MEX S-function implementation of Fractional States-Space Sys-
tem.
  * fsim.dll - compiled fsim.c function.
  * kfc.c - C-MEX S-function implementation of Fractional Kalman Filter
  * kfc.dll - compiled kfc.c function.
  * fss.mdl - Simulink library of the toolkit.
  * sblinks.m - data for Simulink Library Browser.
  * examples\ex1.mdl - Example of systems simulation.
  * examples\ex2.mdl - Example of state feedback control.

After installation the FSST should be seen in Simulink Library Browser. The
toolkit two blocks:

  * "Fractional Order States-Space System" described in Section 2
  * "Fractional Kalman Filter" described in Section 3

The Toolkit s-function is written in C in order to allow using them in Real
Time Workshop and generate code for eg. DS1104 card.
2 Fractional order states-space system s-function

The fractional order states-space system is defined as following (to more detail see [5]):

\[ \Delta^\gamma x_{k+1} = A_d x_k + B u_k \quad (1) \]

\[ x_{k+1} = \Delta^\gamma x_{k+1} - \sum_{j=1}^{k+1} (-1)^j \gamma x_{k-j+1} \quad (2) \]

\[ y_k = C x_k \quad (3) \]

where:

\[ \Upsilon_k = \text{diag} \left[ \begin{pmatrix} n_1 \\ k \\ \vdots \\ n_N \\ k \end{pmatrix} \right] \]

\[ \Delta^\gamma x_{k+1} = \begin{bmatrix} \Delta^{n_1} x_{1,k+1} \\ \vdots \\ \Delta^{n_N} x_{N,k+1} \end{bmatrix} \]

and \( n_1, \ldots, n_N \) are degrees of system equations.

For real-time simulation there is a requirement of limiting the number of samples in sum operation. It was done by implemented the circular buffer with limited number of samples, given by parameter \( Nbuf \).

the \( \binom{n}{j} \) symbol is evaluated as:

\[ \binom{n}{j} = \begin{cases} 1 & \text{for } j = 0 \\ \frac{n(n-1)\ldots(n-j+1)}{j!} & \text{for } j > 0 \end{cases} \quad (4) \]

what for \( j > 0 \) can be counted recursively as:

\[ \binom{n}{j} = \frac{n}{1} \frac{n-1}{2} \ldots \frac{n-j+1}{j} \quad (5) \]

using this form there is no need to count factorial of very high numbers what is limited by type of used variables.

The S-function fsim have following parameters in turn \( A, B, C, N, Ts, Nbuf \).

\( A, B, C, N \) are system matrices where \( A \) is size \( Nx,Nx \), \( B \) is \( Nx, Nu \), \( C \) \( Ny, Nx \), \( N \) \( Nx, 1 \)

\( Ts \) is a sample time

\( Nbuf \) is a width of a circular buffer of past states vectors.

where \( Nx \)-number of states, \( Nu \)-number of inputs, \( Ny \)-number of outputs

3 Fractional Kalman Filter s-function

For system defined in Section 2 the Fractional Kalman Filter is defines as follows (for more detail see [3]):

\[ \]
\[
\begin{align*}
\Delta^T \hat{x}_{k+1} &= A_d \hat{x}_k + B u_k \\
\hat{x}_{k+1} &= \Delta^T \hat{x}_{k+1} + \sum_{j=1}^{k+1} (-1)^j \Upsilon_j \hat{x}_{k-j+1} \\
\hat{P}_k &= (A_d + \Upsilon_1) P_{k-1} (A_d + \Upsilon_1)^T + \\
&\quad + Q_{k-1} + \sum_{j=2}^{k} \Upsilon_j P_{k-j} \Upsilon_j^T \\
K_k &= \hat{P}_k C^T (C \hat{P}_k C^T + R_k)^{-1} \\
\hat{x}_k &= \hat{x}_k + K_k (y_k - C \hat{x}_k) \\
P_k &= (I - K_k C) \hat{P}_k
\end{align*}
\]

where: \( R \) is a covariance matrix of output noise \( \nu \) and \( Q \) is a covariance matrix of system noise \( \omega \). Both of that noises are assumed to be independent and with zero expected value.

The S-function \texttt{fsim} have following parameters in turn \( A, B, C, N, P, Q, R, x_0, Ts, Nbuf \).

\( A, B, C, N \) are system matrices where \( A \) is size \( N_x N_x, B \) is \( N_x N_u \) \( C \) \( N_y N_x \) \( N \) \( N_x, 1 \)

\( P, Q, R, x_0 \) are FKF matrices where \( P \) is size \( N_x N_x \) \( Q N_x N_x \) \( R N_y N_y \) \( x_0 \) \( N_x, 1 \)

\( Ts \) is a sample time

\( Nbuf \) is a width of a circular buffer of past states vectors and past covariance matrices.

The number of system output (rows number of \( C \) matrix) is limited to 1 (in future versions will not be limited).

4 Examples

4.1 Example 1: Fractional State-space systems simulation

The aim of this example is to show dependency of the output of the fractional system to the circular buffer length \( (Nbuf) \). The system used in example is defined by following matrices:

\[
A_d = \begin{bmatrix} 0 & 1 \\ -0.1 & -0.2 \end{bmatrix}, \quad N = \begin{bmatrix} 0.7 & 1.2 \\ 1 & 2 \end{bmatrix} \quad (6)
\]

\[
B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad C = \begin{bmatrix} 0.1 & 0.3 \end{bmatrix} \quad (7)
\]

The system is simulated for different lengths of the circular buffer \( (Nbuf = 10, 20, 50, 100, 300) \). Results are shown in Fig. 1.
4.2 Example 2: Fractional Order States-Space System states feedback control

This example shows the use of states estimation doing by FKF to states feedback control (more detail will be presented at [6]).

The system used in example is defined by following matrices:

\[
A_d = \begin{bmatrix} 0 & 1 \\ -0.35 & -0.1 \end{bmatrix}, \quad N = \begin{bmatrix} 0.8 \\ 0.7 \end{bmatrix}
\]

\[B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad C = \begin{bmatrix} 0.3 & 0.4 \end{bmatrix}\]

(8)

(9)

The control law is defined as:

\[u = u_{ref} - Kx\]

(10)

where

\[K = \begin{bmatrix} -0.15 & 0.3 \end{bmatrix}\]

(11)

The reference system, for that control law, has a matrix A as follows:

\[
A_d = \begin{bmatrix} 0 & 1 \\ -0.2 & -0.4 \end{bmatrix}
\]

(12)
This example shows the use of Fractional Kalman Filter for states feedback control system. The Scope shows original and estimated states variables, which are used by regulator. Scope1 presents outputs of the reference object and control object.

Figure 2: Close-loop feedback control example scheme

Figure 3: Results of states estimation in close-loop feedback control
Figure 4: Results of outputs in close-loop feedback control

5 License

The Fractional States-Space Toolkit is free-ware for non-commercial usage.

References


[6] A. Dzieliński D. Sierociuk "Fractional order discrete state-space system estimation, identification and control" 44th IEEE Conference on Decision and Control, Seville, 12-15 December 2005 (paper has been send to this conference)